Math Virtual Learning

Precalculus with Trigonometry

May 5, 2020

Precalculus with Trigonometry
Lesson: May 5th, 2020

Objective/Learning Target:
Students will verify Trigonometric Identities using the Half-Angle Identities

Let's Get Started:

Watch the video below to see how to verify half-angle identities.

Watch Video: [Master How to Verify a trigonometric identity using half angle formulas]

Recall these formulas while watching the video.

Half Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad \text{or} \quad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} \quad \text{or} \quad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \quad \text{or} \quad \frac{\sin x}{1+\cos x} \quad \text{or} \quad \frac{1-\cos x}{\sin x}$$

Example #1:

Verify the given identity.

$$\csc^2\left(\frac{\theta}{2}\right) = \frac{2}{1-\cos\theta}$$

$$\frac{\mathbf{1}}{\mathbf{\sin^2\left(\frac{\theta}{2}\right)}} = \frac{2}{1-\cos\theta}$$ csc is the reciprocal of sin

$$\frac{1}{\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2} = \frac{2}{1-\cos\theta}$$ squaring the half-angle formula for sin

$$\frac{1}{\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2} = \frac{2}{1-\cos\theta}$$ square root and the square cancel out

$$\frac{\left(\frac{2}{1-\cos\theta}\right)1}{\left(\frac{2}{1-\cos\theta}\right)\frac{1-\cos\theta}{2}} = \frac{2}{1-\cos\theta}$$ when dividing a fraction, multiply by the reciprocal

$$\frac{2}{1-\cos\theta} = \frac{2}{1-\cos\theta}$$

Example #2:

Verify the given identity.

$$\cot^2\left(\frac{v}{2}\right) = \frac{\sec v + 1}{\sec v - 1}$$

$$\frac{1}{\tan^2\left(\frac{v}{2}\right)} = \frac{\sec v+1}{\sec v-1}$$ tan is the reciprocal of cot

$$\frac{1}{\left(\pm\sqrt{\frac{1-\cos v}{1+\cos v}}\right)^2} = \frac{\sec v+1}{\sec v-1}$$ squaring the half-angle formula for tan

$$\frac{1}{\left(\pm\sqrt{\frac{1-\cos v}{1+\cos v}}\right)^2} = \frac{\sec v+1}{\sec v-1}$$ square root and the square cancel out

$$\frac{\left(\frac{1+\cos v}{1-\cos v}\right)1}{\left(\frac{1+\cos v}{1-\cos v}\right)\frac{1-\cos v}{1+\cos v}} = \frac{\sec v+1}{\sec v-1}$$ when dividing a fraction, multiply by the reciprocal

$$\frac{\sec v(1+\cos v)}{\sec v(1-\cos v)} = \frac{\sec v+1}{\sec v-1}$$ multiply the numerator and denominator by the same amount

$$\frac{\sec v+\cos v \sec v}{\sec v-\cos v \sec v} = \frac{\sec v+1}{\sec v-1}$$ since cos and sec are inverses, they multiply to equal 1

$$\frac{\sec v + 1}{\sec v - 1} = \frac{\sec v + 1}{\sec v - 1}$$

Practice

On a separate piece of paper, use the Half-Angle Identities to verify the following equations.

1. $\sin^2\left(\frac{x}{2}\right) = \frac{\csc x - \cot x}{2\csc x}$

2. $\cos^2\left(\frac{x}{2}\right) = \frac{\sin x + \tan x}{2\tan x}$

3. $\tan\left(\frac{\theta}{2}\right) = \csc\theta - \cot\theta$

4. $\frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)} = \cos\theta$

Practice - **ANSWERS**

On a separate piece of paper, use the Half-Angle Identities to verify the following equations.

Worked out Solution for Questions 1 and 2:

Video 1: Verifying Trigonometric Identities Using Half Angle Formulas

1. $\sin^2\left(\frac{x}{2}\right) = \frac{\csc x - \cot x}{2\csc x}$

Solution at: **Start of Video 1**

2. $\cos^2\left(\frac{x}{2}\right) = \frac{\sin x + \tan x}{2\tan x}$

Solution at: **1:51 of Video 1**

Worked out Solution for Questions 3 and 4

Video 2: How to verify half-angle identities for tangent

3. $\tan\left(\frac{\theta}{2}\right) = \csc\theta - \cot\theta$

Solution at: **Start of Video 2**

4. $\frac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)} = \cos\theta$

Solution at: **0:52 of Video 2**

Additional Resource Videos:

How to verify half-angle identities. Sec, csc, cot

Additional Practice:

Half-Angle Formulas (with solutions)

Examples and Exercises start about ⅔ of the way down the page.

Double & Half Angle Formulas Practice (no solutions)

Examples 6 & 7 on page 613. Exercises 59-68 on page 615